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# QCD at finite temperature and density

Kazuyuki Kanaya, for the WHOT-QCD Collaboration

WHOT-QCD Collaboration: S. Aoki, S.Ejiri, T.Hatsuda, N.Ishii, KK, Y.Maezawa, Y.Nakagawa, H.Ohno, K.Okuno, H.Saito, N.Ukita, T.Umeda, S.Yoshida, ...

# Phase structure of QCD

QCD expected to have a rich phase structure at finite T 's and  $\mu$ 's.



<= Theoretical inputs directly from the 1st principles of QCD important.

# Our motivations

Theoretical inputs directly from the 1st principles of QCD important.



properties of the matter in each phase: screening length, susceptibilities, ...

preciselines / critical points / ...

We study them using improved Wilson quarks, which are guaranteed to have the correct continuum limit.

WHOT-QCD Collaboration: S. Aoki, S.Ejiri, T.Hatsuda, N.Ishii, KK, Y.Maezawa, Y.Nakagawa, H.Ohno, K.Okuno, H.Saito, N.Ukita, T.Umeda, S.Yoshida, ...

# Our main results since 2008

#### • EOS at $\mu \neq 0$ with dynamical Wilson quarks

Phys. Rev. D 82 (2010) ref.014508, "Equation of State and Heavy-Quark Free Energy at Finite Temperature and Density in Two Flavor Lattice QCD with Wilson Quark Action ", S. Ejiri, Y. Maezawa, N. Ukita, S. Aoki, T. Hatsuda, N. Ishii, K. Kanaya, T. Umeda

#### • EOS with fixed-scale approach

Phys. Rev. D 79 (2009) ref.051501(R), "Fixed Scale Approach to Equation of State in Lattice QCD", T. Umeda, S. Ejiri, S. Aoki, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno

Phys. Rev. D 85 (2012) ref.094508, "Equation of state in 2+1 flavor QCD with improved Wilson quarks by the fixed scale approach", T. Umeda, S. Aoki, S. Ejiri, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno

#### Search for the critical point with histogram method

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Phys. Rev. D (2014) in press, "Histograms in heavy-quark QCD at finite temperature and density", H. Saito, S. Ejiri, S. Aoki, K. Kanaya, Y. Nakagawa, H. Ohno, K. Okuno, T. Umeda

#### • Q- $\overleftarrow{Q}$ interaction and screening masses at finite T and $\mu$

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**Prog.** Theor. Phys. 128 (2012), "Application of fixed scale approach to static quark free energies in quenched and 2 + 1 flavor lattice QCD with improved Wilson quark action", Y. Maezawa, T. Umeda, S. Aoki, S. Ejiri, T. Hatsuda, K. Kanaya and H. Ohno

#### Charmonium dissociation with variational method

Phys. Rev. D 84 (2011) ref.094504, "Charmonium spectral functions with the variational method in zero and finite temperature lattice QCD", H. Ohno, S. Aoki, S. Ejiri, K. Kanaya, Y. Maezawa, H. Saito and T. Umeda

# Lattice QCD at $\mu \neq 0$

- $\bigcirc \ LQCD \text{ at } \mu \neq 0 \qquad U_4 \longrightarrow \left\{ \begin{array}{ll} U_4 e^{\mu a} & \cdots & \text{positive } t \text{ direction} \\ U_4 e^{-\mu a} & \cdots & \text{nagative } t \text{ direction} \end{array} \right.$ 
  - $= > [\det M(\mu)]^* = \det M(-\mu^*) \neq \det M(\mu)$
  - => MC based on importance sampling with det*M* not justified
- Sign problem (complex phase problem) phase-quenched simulation by det  $M \rightarrow Idet MI$ , and handling the phase in the measurement

$$\begin{array}{l} \text{``reweighting''} \quad \langle \mathcal{O} \rangle = \frac{\int d\Phi \, \mathcal{O} \, e^{-S}}{\int d\Phi \, e^{-S}} \\ = \frac{\int d\Phi \, \mathcal{O} e^{-\Delta S} \, e^{-S + \Delta S}}{\int d\Phi \, e^{-\Delta S} \, e^{-S + \Delta S}} = \frac{\langle \mathcal{O} e^{-\Delta S} \rangle_{S - \Delta S}}{\langle e^{-\Delta S} \rangle_{S - \Delta S}} \end{array}$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int DU \mathcal{O} \left[ \det M \right]^{N_F} e^{-S_g} = \frac{\langle \mathcal{O} e^{iN_F \theta} \rangle_{\text{p.q.}}}{\langle e^{iN_F \theta} \rangle_{\text{p.q.}}}, \quad \det M = \left| \det M \right| e^{i\theta}$$

=> Exponentially high statistic required when  $\theta$  fluctuates a lot (<= large  $\mu$ ).

# Lattice QCD at $\mu \neq 0$

- $\bigcirc$  Techniques for small  $\mu/T$ 
  - Taylor expansion around  $\mu = 0$
  - multi-parameter reweighting
  - + imaginary  $\mu$  (analytic continuation to real  $\mu$ )
  - canonical ensemble
  - complex Langevin
  - ✦ Lefschetz thimble etc. etc.
    - combination of them & other techniques to extend the range of applicability

Our previous study at  $\mu=0$ :  $N_F = 2$  QCD,  $N_t=4$ , (6)



#### $\mu \neq 0$ by the Taylor expansion method:

Observables  $\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z} \equiv \omega, \qquad \frac{n_f}{T^3} = \frac{1}{VT^3} \frac{\partial \ln \mathcal{Z}}{\partial (\mu_f/T)} = \frac{\partial (p/T^4)}{\partial (\mu_f/T)}, \qquad (f = u, d)$  $\frac{\chi_q}{T^2} = \left(\frac{\partial}{\partial(\mu_u/T)} + \frac{\partial}{\partial(\mu_d/T)}\right) \frac{n_u + n_d}{T^3} \quad \frac{\chi_I}{T^2} = \left(\frac{\partial}{\partial(\mu_u/T)} - \frac{\partial}{\partial(\mu_d/T)}\right) \frac{n_u - n_d}{T^3}$ Taylor expansion in  $\mu_u = \mu_d (= \mu_q)$  at  $\mu_u = \mu_d = 0$  $\frac{p}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu_q}{T}\right)^n, \qquad c_n(T) = \frac{1}{n!} \frac{N_t^3}{N_s^3} \left. \frac{\partial^n \ln \mathcal{Z}}{\partial (\mu_q/T)^n} \right|_{\mu_s = 0}$  $\frac{\chi_q(T,\mu_q)}{T^2} = 2c_2 + 12c_4 \left(\frac{\mu_q}{T}\right)^2 + \cdots \qquad c_2 = \frac{N_t}{2N^3} \mathcal{A}_2, \quad c_4 = \frac{1}{4!N^3N_t} (\mathcal{A}_4 - 3\mathcal{A}_2^2)$  $\mathcal{A}_2 = \langle \mathcal{D}_2 \rangle + \langle \mathcal{D}_1^2 \rangle, \quad \mathcal{A}_4 = \langle \mathcal{D}_4 \rangle + 4 \langle \mathcal{D}_3 \mathcal{D}_1 \rangle + 3 \langle \mathcal{D}_2^2 \rangle + 6 \langle \mathcal{D}_2 \mathcal{D}_1^2 \rangle + \langle \mathcal{D}_1^4 \rangle$  $\mathcal{D}_n = N_f \frac{\partial^n \ln \det M}{\partial \mu^n} \qquad \qquad \mathcal{D}_1 = N_f \operatorname{tr} \left( M^{-1} \frac{\partial M}{\partial \mu} \right)$  $\mathcal{D}_2 = N_f \left[ \operatorname{tr} \left( M^{-1} \frac{\partial^2 M}{\partial \mu^2} \right) - \operatorname{tr} \left( M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} \right) \right]$  $\mu \equiv \mu_q a$ 

Phys. Rev. D 82 (2010) ref.014508, "Equation of State and Heavy-Quark Free Energy at Finite Temperature and Density in Two Flavor Lattice QCD with Wilson Quark Action ", S. Ejiri, Y. Maezawa, N. Ukita, S. Aoki, T. Hatsuda, N. Ishii, K. Kanaya, T. Umeda

Noise method for traces in the Taylor coefficients

$$\frac{1}{N_{\text{noise}}} \sum_{i} \eta_{i,\alpha} \eta_{i,\beta}^* \approx \delta_{\alpha,\beta} \qquad \text{tr} \left(\frac{\partial^n M}{\partial^n \mu} M^{-1}\right) \approx \frac{1}{N_{\text{noise}}} \sum_{i} \eta_i^{\dagger} \frac{\partial^n M}{\partial^n \mu} X_i; \quad X_i = M^{-1} \eta_i$$

For Wilson quarks, we generate independent  $\eta$  for each color and spin. Error from  $D_1$  turned out to be dominating in the results. => about 100 times larger  $N_{\text{noise}}$  for  $D_1$ .





 $\lambda \chi q$ 

 $\blacktriangleright \chi_I$ 

FIG. 14: Quark number susceptibility at finite  $\mu_q$  for  $m_{\rm PS}/m_{\rm V} = 0.65$  (left) and 0.80 (right).





Suggest critical pt. at finite  $\mu$ , which is insensitive to the iso-spin number.

To further improve the calculation

#### A hybrid Taylor+reweighting method

Allton et al., PRD 66, 074507 ('02) Ejiri, PRD 77, 014508 ('08)

Reweight the grand canonical partition function from  $\mu=0$ :

$$\mathcal{Z}(T,\mu_q) = \mathcal{Z}(T,0) \left\langle \left(\frac{\det M(\mu)}{\det M(0)}\right)^{N_{\rm f}} \right\rangle_{(\mu_q=0)} \equiv \mathcal{Z}(T,0) \left\langle e^{F(\mu)} e^{i\theta(\mu)} \right\rangle_{(\mu_q=0)}$$

and Taylor-expand the terms in exp:

Truncate the expansions up to  $D_4$ 

\* Identical to the truncated Taylor expansion up to the 4th order,

but contain a part of higher orders through the exponential function.

\* Exact for free QGP, in which  $D_n=0$  for n>4, => The truncation will be OK at high T.

#### $\blacktriangleright$ Gaussian approximation for $\theta$ distribution at small $\mu$



Ejiri, PRD 77, 014508 ('08)

Sign problem avoided at small  $\mu$  => See discussions later.



FIG. 24: Quark number susceptibility for each  $\mu_q/T$  at  $m_{\rm PS}/m_{\rm V} = 0.65$  (left) and 0.80 (right).

▶p(µ)-p(0)



The first EOS at  $\mu \neq 0$  with Wilson-type quarks.

 $R = 1, 8, 6, 3^*$ 

heavy quark free energies at  $\mu \neq 0$ 

$$V^{R}(r, T, \mu_{q}) = v_{0}^{R} + v_{1}^{R} \left(\frac{\mu_{q}}{T}\right) + v_{2}^{R} \left(\frac{\mu_{q}}{T}\right)^{2} + O(\mu^{3})$$



We want to extend the studies to  $N_F=2+1$ , larger  $N_t$ , and smaller  $m_q$ More improvements / developments of the method needed.

We note: a large fraction of the cost due to T = 0 simulations

Determination of LCP, scale, beta functions, etc.

T = 0 subtractions for renormalization (needed at all the simulation points!)

#### Fixed scale approach with a T-integration method.

Pros and cons: T=0 simulation costs largely removable

A common T=0 simulation enough for all T=0 subtractions. We can even borrow publicly available configurations on ILDG.
Automatically on a LCP w/o fine tuning.

Limited resolution in T => need to check if it works on isotropic lattices.

Other properties are complementary to the fixed-Nt approach.



#### A test in quenched QCD => looks fine.

Phys. Rev. D 79 (2009) ref.051501(R), "Fixed Scale Approach to Equation of State in Lattice QCD", T. Umeda, S. Ejiri, S. Aoki, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno



Besides understandable deviations, results consistent with each other

- T-interpolation under control on the isotropic lattice
- computation costs much reduced

#### Study in Nf=2+1 QCD

Phys. Rev. D 85 (2012) ref.094508, "Equation of state in 2+1 flavor QCD with improved Wilson quarks by the fixed scale approach", T. Umeda, S. Aoki, S. Ejiri, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno

- T=0 simulation: on 28<sup>3</sup> x 56 by CP-PACS/JLQCD Phys. Rev. D78 (2008) 011502
  - RG-improved Iwasaki glue + NP-improved Wilson quarks
  - $\beta$ =2.05,  $\kappa_{ud}$ =0.1356,  $\kappa_s$ =0.1351
  - V~(2 fm)<sup>3</sup>, a~0.07 fm,
  - configurations available on the ILDG/JLDG
- **T**>0 simulations: on  $32^3 \times N_t$  (N<sub>t</sub>=4, 6, ..., 14, 16) lattices



**RHMC algorithm, same parameters as T=0 simulation** At the lightest point on the finest lattice among 30 simulation points of CP-PACS/JLQCD.





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#### roughly consistent with EOS from highly improved stag. quarks

FIG. 8 (color online). Trace anomaly  $(\epsilon - 3p)/T^4$ , energy density  $\epsilon/T^4$ , and pressure  $3p/T^4$  in 2 + 1 flavor QCD. The thin and thick vertical bars represent statistic and systematic errors, respectively. The curves are drawn by the Akima spline interpolation.

The first Nf=2+1 EOS with Wilson-type quarks.

To extend the study to  $\mu \neq 0$ , we first have to clarify the phase structure. To efficiently detect the phase, we developed a method based on the histogram.

Histogram and effective potential of observables

$$\begin{split} w(\mathcal{O}_{1},\mathcal{O}_{2},\cdots;\beta,m,\mu) &\stackrel{\text{def.}}{=} \int DU \prod_{i} \delta(\hat{\mathcal{O}}_{i}[U] - \mathcal{O}_{i}) \left[\det M(m,\mu)\right]^{N_{F}} e^{-S_{g}(\beta)} \\ Z_{(\beta,m,\mu)} &= \int w(\mathcal{O}_{1},\cdots;\beta,m,\mu) \prod_{i} d\mathcal{O}_{i} \qquad \left\langle f(\hat{\mathcal{O}}_{1},\cdots)\right\rangle_{(\beta,m,\mu)} = \frac{1}{Z_{(\beta,m,\mu)}} \int f(\mathcal{O}_{1},\cdots) w(\mathcal{O}_{1},\cdots;\beta,m,\mu) \prod_{i} d\mathcal{O}_{i} \\ V_{\text{eff}}(\hat{\mathcal{O}}_{1},\cdots;\beta,m,\mu) \stackrel{\text{def.}}{=} -\ln w(\mathcal{O}_{1},\cdots;\beta,m,\mu) \end{split}$$

Choosing we can detect the phase transition through



We need reliable Veff in a wide range of  ${\cal O}$  around the transition pt.

<= reweighing technique & appropriate choice of  ${\cal O}$ 



Phys. Rev. D 84 (2011) ref.054502, "Phase structure of finite temperature QCD in the heavy quark region", H. Saito, S. Ejiri, S. Aoki, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno, T. Umeda

#### <u>Search for the critical point with</u> histogram method $\frac{2nd}{O(4)}(?)$ crossove Phys poin

#### Heavy quark QCD at $\mu$ =0:

Heavy quark limit = SU(3) YM theory with 1 st order deconf. trans. Reweight from the heavy quark limit, using the hopping param. expansion.

$$\left[\frac{\det M(\kappa,\mu)}{\det M(0,0)}\right]^{N_F} = \exp\left[N_F\left\{288N_{\rm site}\kappa^4\hat{P} + 12\cdot 2^{N_t}N_s^3\kappa^{N_t}\left(\cosh(\mu/T)\hat{\Omega}_{\rm R} + i\sinh(\mu/T)\hat{\Omega}_{\rm I}\right) + \cdots\right\}\right]$$

2 QCD

 $m_s$ 

SU(3) Y

 $N_{E} = 2 + 1 \text{ QCD}$ 



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#### Heavy quark QCD at $\mu \neq 0$ : Reweight to $\mu \neq 0$ $V_{\text{eff}}(\mathcal{O}; \beta, \kappa, \mu) = V_{\text{eff}}(\mathcal{O}; \beta, 0, 0) - \ln \left\langle \left[ \frac{\det M(\kappa, \mu)}{\det M(0, 0)} \right]^{N_F} \right\rangle_{\mathcal{O}; \beta, 0, 0}$ $\left[ \frac{\det M(\kappa, \mu)}{\det M(0, 0)} \right]^{N_F} = \exp \left[ N_F \left\{ 288N_{\text{site}} \kappa^4 \hat{P} + 12 \cdot 2^{N_t} N_s^3 \kappa^{N_t} \left( \cosh(\mu/T) \hat{\Omega}_{\text{R}} + i \sinh(\mu/T) \hat{\Omega}_{\text{I}} \right) + \cdots \right\} \right]$ Choose $V_{\text{eff}}(\Omega_{\text{R}}; \beta, \kappa, \mu) = V_{\text{eff}}(\Omega_{\text{R}}; \beta^*, 0, 0) - 12 \cdot 2^{N_t} N_F N_s^3 \kappa^{N_t} \cosh(\mu/T) \Omega_{\text{R}} - \ln \left\langle e^{i\hat{\theta}} \right\rangle_{\Omega_{\text{R}}; \beta, 0, 0}$

 $\hat{\theta} = 12 \cdot 2^{N_t} N_F N_s^3 \kappa^{N_t} \sinh(\mu/T) \,\hat{\Omega}_{\mathrm{I}}$ 

Phase-quenched QCD is simple:

Critical line in the phase-quenched QCD is just given by

$$\kappa_{\rm cp}^{\rm p.q.}(\mu) = \kappa_{\rm cp}(\mu=0) \cdot [\cosh(\mu/T)]^{-1/N_t}$$

Phase-quenched QCD

 $V_{\rm eff}(\Omega_{\rm R};\beta,\kappa,\mu) = V_{\rm eff}(\Omega_{\rm R};\beta^*,0,0) - 12 \cdot 2^{N_t} N_F N_s^3 \kappa^{N_t} \cosh(\mu/T) \Omega_{\rm R} - \ln\left\langle e^{i\hat{\theta}} \right\rangle_{\Omega_{\rm R};\beta,0,0}$ 

To study the effects of the phase term,

Cumulant expansion

S. Ejiri, PRD 77, 014508 ('08); WHOT, PRD 82, 014508 ('10)

$$\begin{split} \langle e^{i\hat{\theta}} \rangle &= \exp\left[i\langle\hat{\theta}\rangle_c - \frac{1}{2!}\langle\hat{\theta}^2\rangle_c - \frac{i}{3!}\langle\hat{\theta}^3\rangle_c + \frac{1}{4!}\langle\hat{\theta}^4\rangle_c + \cdots\right] \\ \langle\hat{\theta}\rangle_c &= \langle\hat{\theta}\rangle, \ \langle\hat{\theta}^2\rangle_c = \langle\hat{\theta}^2\rangle - \langle\hat{\theta}\rangle^2, \ \langle\hat{\theta}^3\rangle_c = \langle\hat{\theta}^3\rangle - 3\langle\hat{\theta}^2\rangle\langle\hat{\theta}\rangle + 2\langle\hat{\theta}\rangle^3, \ \cdots \end{split}$$

• Odd terms vanish due to the symmetry under  $\mu => \langle e^{i\hat{\theta}} \rangle$  positive definite. => Sign problem resolved if the expansion converges.



FIG. 9: Exponent of the average phase factor,  $-\ln \langle e^{i\hat{\theta}} \rangle$ , compared with the contributions from the 2<sup>nd</sup>, 4<sup>th</sup> and 6<sup>th</sup> order cumulants. The expectation values are calculated at  $\beta^* = 5.69$  and  $\kappa^4 \sinh(\mu/T) \approx 0.00002$  (left) or 0.00005 (right) in  $N_{\rm f} = 2$  QCD with fixed  $\Omega_{\rm R}$ . In both cases,  $-\ln \langle e^{i\hat{\theta}} \rangle$  is almost indistinguishable with  $\langle \hat{\theta}^2 \rangle_c / 2!$ .

#### Heavy quark QCD at $\mu \neq 0$ :

We can now reliably evaluate  $\langle e^{i\hat{\theta}} \rangle$  by the cumulant expansion around the crit. line.

=> The effects turned out to be quite small on the crit. line at all  $\mu$ .

=> The critical line well estimated by the phase-quenched approx.



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# $Q-\overline{Q}$ interaction and screening masses at finite T and $\mu$

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► Heavy quark free energy at  $T > T_c$   $F^1(r,T) = -T \ln \langle \operatorname{Tr} \Omega^{\dagger}(\mathbf{x}) \Omega(\mathbf{y}) \rangle$  in the Coulomb gauge

V(r, T = 0)T=0 data by CP-PACS+JLQCD  $N_f = 2 + 1$ r = 232 MeV **1**\_₩ + = 279 MeV $2F_Q = -2T \ln \langle \mathrm{Tr} \Omega \rangle$ T = 348 MeVT = 464 MeV $2 \times single quark free energy$ T = 696<sup>F1</sup>(r,T) [GeV] 2  $F^{1} \longrightarrow 2F_{Q}$  $F^{|} \longrightarrow V(r) = F^{|}(T=0)$ at long distances. at short distances. i.e. deconfinement at IR 0.2 0.40.6 0.8 i.e. T-insensitivity at UV." r [fm] **No vertical adjustment needed** in the fixed scale approach.

(cf.) In the fixed-Nt approach,  $F^{1} \rightarrow V(r)$  is used as an input to adjust the constant term of  $F^{1}$ .

### <u>Charmonium dissociation with</u> <u>variational method</u>

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Computation of the spectral function (SpF) on the lattice is an ill-posed problem. Conventional estimation using MEM is always heuristic.

We define an effective SpF in terms of the correlation matrix between a series of smeared operators. This eff. SpF tends to the correct SpF on the lattice at large distances, and the systematic error due to finite distance can be reduced by increasing the number of operators and by a judicious choice of the set of operators.

#### A test with free quarks



With dynamical quarks



Analytic results for low-lying poles and their width well reproduced by increasing # of sources.

Ground state consistent with experiment as well as the MEM result. Ist exited state deviates from the MEM estimation and gets closer to the experiment with increasing # of sources.

# Our main results since 2008

#### • EOS at $\mu \neq 0$ with dynamical Wilson quarks

Cumulant expansion useful at small  $\mu$ . The first EOS at  $\mu \neq 0$  with Wilson-type quarks.

• EOS with fixed-scale approach

Fixed-scale approach with T-integration method works. The first Nf=2+1 EOS with Wilson-type quarks.

#### Search for the critical point with histogram method

Histogram + reweighting powerful for the phase structure. Test in heavy quark QCD.

#### ${}^{\bullet}$ Q- $\overline{Q}$ interaction and screening masses at finite T and $\mu$

Q- $\overline{Q}$  int. weaker at  $\mu \neq 0$ , Q-Q int. stronger at  $\mu \neq 0$ . Fixed-scale approach allows us a direct test of T-dependence.

#### Charmonium dissociation with variational method

A solid method to compute spectral function on the lattice. Works well for low-lying states (pole position and width).

# Plans for the next stage

#### Nf=2+1 EOS at the physical point

Extend the EOS study at  $\mu$ =0 towards the physical point on a fine lattice.

- \* Fixed-scale approach making use of T=0 PACS-CS or HAL configurations.
- \* Need beta functions <= reweighing technique?

#### Critical point at µ≠0

Extend the phase structure study to light quark QCD.

- \* Histogram method with dynamical quarks.
- \* Can detect the critical point?
  - => See below.



#### Scaling properties at $\mu=0$ and $\neq 0$

- \* Tricritical point, curvature of the pseudo critical line, ...
- \* O(4)? or something else?

#### QCD with light quarks:



#### QCD with light quarks:

Our strategy:

\* Phase-quenched simulation with  $|detM|^{Nf} e^{-Sg}$ 

\* Cumulant expansion for the phase  $e^{i\theta}$  with  $\theta(\mu) = N_f \int_0^{\mu/T} \Im \left[ \frac{\partial (\ln \det M(\mu))}{\partial (\mu/T)} \right]_{\bar{\mu}} d\left(\frac{\bar{\mu}}{T}\right)$ 

\* To cover a wide range of V<sub>eff</sub> by another reweighting, we choose

 $P = -S_g/6\beta N_{site} \approx glue energy$  $F = N_f \ln \left[ det M(\mu)/det M(0) \right] \approx quark energy$ 

as O's for  $V_{eff}$ 

$$Z(\beta,\mu) = \int \mathscr{D}U \, e^{i\theta(\mu)} \left|\det M(\mu)\right|^{N_{\rm f}} e^{6\beta N_{\rm site}P}$$

We again find Gaussian dominance at small  $\mu/T$ .

$$e^{i\theta}\Big\rangle_{P,F} \approx \exp\left[-\frac{1}{2}\left\langle\theta^2\right\rangle\right]$$

Can we reach the crit. point?



0.07





# thank you